



# Radial magnetic field measurements with a Hall probe device in the muon (g-2) storage ring magnet at BNL

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## Abstract

A Hall probe device has been built to measure the radial component of the magnetic field in the muon (g-2) storage ring at Brookhaven National Laboratory. The ultraprecise (g-2) magnet provides a dominantly vertical magnetic field of about 1.45 T. In order to limit the vertical shift of the muon orbit, the average radial field component should be no more than  $5 \times 10^{-5}$  of the vertical field. Our measurements with the Hall probe device achieved an accuracy of  $1 \times 10^{-5}$ , which is one of the most precise measurements with Hall probes. This provides adequate accuracy for shimming and control of the radial field. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Hall probe; Magnetic field measurement; Ultraprecise superferic magnet

## 1. Introduction

An ultraprecise superferic magnet [1], 14 m in diameter, was built for the muon (g-2) experiment at Brookhaven National Laboratory. The principal quantities to measure in the experiment are the

frequency of (g-2) oscillations and the average magnetic field. This implies stringent requirements on the homogeneity of the magnetic field as well as on the precision of its measurement and control system, which are based on pulsed NMR probes [2]. In addition to the measurements with NMR probes, which determine very accurately the absolute value of the magnetic field but are insensitive to its direction, a number of measurements were done with Hall probes [3], the most

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important being the measurement of the radial component of the magnetic field in the muon storage ring.

In the weak focusing (g-2) storage ring, horizontal focusing is provided by the magnetic field and vertical focusing by an electrostatic quadrupole field with an average field gradient of  $8.4 \times 10^6 \text{ V/m}^2$  in the center of the aperture. We use a cylindrical coordinate system  $R$ ,  $Z$  and  $\Theta$ , where  $R$  is radial coordinate,  $\Theta$  is the azimuthal angle in the storage ring, and  $Z$  is the vertical coordinate. A nonvanishing average radial magnetic field  $B_R$  shifts the muon orbit vertically to a point where the magnetic force  $evB_R$  ( $v \approx c$ ) is compensated by the force  $eE_Z$  from the vertical E-field component  $E_Z$ . Thus an average radial magnetic field of  $2.9 \times 10^{-5} \text{ T}$ , or 20 ppm (parts per million) of the main field, shifts the beam vertically by 1 mm. The muon beam aperture determined by collimators is circular with a diameter of 9 cm. The design goal was an average radial field of less than 50 ppm corresponding to a vertical shift of the beam center by less than 2.5 mm. This limit can be expressed in terms of the angle  $\alpha$  of the magnetic field with respect to the radial axis. For a radial magnetic field component of 50 ppm of  $B_Z$ , the angle  $\alpha$  is 50  $\mu\text{rad}$ .

In the following sections we discuss the design of the Hall probe device, its calibration and measurements in a test magnet and in the muon storage ring magnet. Test measurements and a number of cross-checks consistently gave an accuracy for the device of 5–10 ppm, which was quite adequate for the shimming and control of the radial magnetic field in the (g-2) magnet.

## 2. Design of the Hall probe device

The Hall probe device shown in Fig. 1 contains two Hall probes BH-206 and an electrolytic tilt sensor RG33A. The sensitive volume of the Hall probe is a thin  $1.5 \times 3.8 \text{ mm}^2$  rectangular plate with current electrodes along the short sides and voltage outputs from the centers of the long sides of the plate. The Hall probes are installed in the  $Z$ – $\Theta$  plane (so they are sensitive mostly to the radial field), and the electrolytic tilt sensor is mounted along the  $R$ -axis on an aluminum support. The support itself is mounted on an aluminum base (not shown in Fig. 1) in such a way that it can be adjusted within a few degrees from the horizontal in the  $Z$ – $R$  plane and rotated about the vertical  $Z$ -axis by  $180^\circ$ . All elements of the

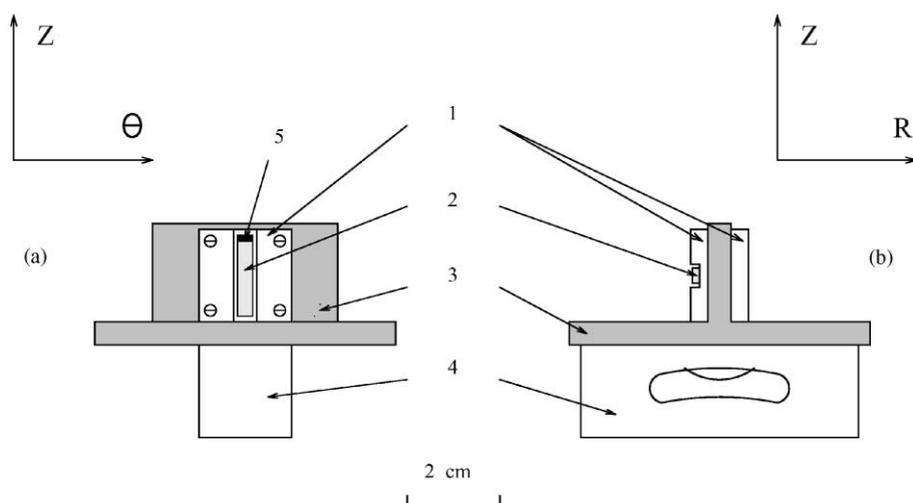


Fig. 1. Hall probe device for the radial magnetic field measurements, front (a) and side (b) views: (1) aluminum plate with Hall probe; (2) Hall probe; (3) aluminum support; (4) tilt sensor; (5) sensitive area of Hall probe.

device and the calibration fixtures are made of nonmagnetic materials.

The principle of operation is the following. First we set the tilt of the aluminum support at some arbitrary angle  $\delta$ , usually close to zero, and record the output voltage  $V$  from the Hall probe. Then we make the  $180^\circ$  rotation, adjust the angle of the support to the same angle  $\delta$  and record another Hall probe voltage,  $V'$ . The angle  $\alpha$  between the magnetic field and the vertical axis in the  $Z$ – $R$  plane is given by

$$\alpha = \frac{1}{2} \frac{V - V'}{V_o} \quad (1)$$

where  $V_o$  is the Hall probe output voltage obtained from a measurement of the main field component of 1.45 T.

Eq. (1) is insensitive to the following sources of systematic errors:

- The Hall probe output voltage has an offset.
- The electrolytic tilt sensor output has an offset.
- The axis of the  $180^\circ$  rotation is not exactly vertical, but tilted in the  $Z$ – $R$  plane.
- The axis of the electrolytic tilt sensor is not exactly orthogonal to the plane of the Hall probes.

In view of the small temperature dependence of the Hall probe characteristics and the short measurement time ( $\sim 5$  min due to relaxation time of the electrolytic liquid in the tilt sensor), no appreciable error arises from temperature variations.

A systematic error due to a rotation by slightly more or less than  $180^\circ$  is of second order in the angular deviation.

A potentially important source of systematic error in the measurement of minor field components is the planar Hall effect. The electric field  $\vec{E}$  in a semiconductor probe in a magnetic field  $\vec{B}$  is given in Ref. [4]

$$\vec{E} = \rho \vec{J} + R_h \vec{B} \times \vec{J} - M \rho \vec{B} \times [\vec{B} \times \vec{J}] \quad (2)$$

where  $\rho$  is the resistivity,  $\vec{J}$  is the current density,  $R_h$  is the Hall magnetoresistivity and  $M$  is the transverse magnetoresistivity. The voltage across the probe can be written as

$$V = k_1 B_\perp I + k_2 \vec{B}_\parallel^2 I \sin 2\psi \quad (3)$$

where  $B_\perp$  is the component of the magnetic field perpendicular to the plane of the Hall probe,  $\vec{B}_\parallel$  is the magnetic field in the plane of the Hall probe and  $\psi$  is the angle between  $\vec{B}_\parallel$  and  $\vec{I}$ . The quantities  $k_1$  and  $k_2$  are the Hall coefficient and the planar Hall coefficient, respectively. In our case, when  $|\vec{B}_\parallel| = 1.45$  T and  $B_\perp \ll |\vec{B}_\parallel|$ , the second term in Eq. (3), which is the planar Hall effect, is typically larger than the direct Hall effect, unless  $\psi$  is close to  $0^\circ$  or  $90^\circ$ .

For most mechanical misalignments the planar Hall effect acts as an additional zero offset and hence is irrelevant. However, if the axis of the  $180^\circ$  rotation is not exactly vertical and has a small component  $\varepsilon$  in the azimuthal direction, Eq. (1) gives a systematic shift by

$$\Delta\alpha = -\frac{1}{2} \frac{k_2 \vec{B}_\parallel^2 I}{V_o} \cos 2\psi \sin 2\varepsilon. \quad (4)$$

To avoid this systematic error, we use two Hall probes with parallel planes, but with orthogonal directions of the Hall currents, in the vertical and horizontal (azimuthal) direction, respectively. If we apply Eq. (1) separately to both Hall probes, which are chosen to have the same planar Hall coefficient  $k_2$ , and then take an average, the net result will not be affected by this source of systematic error:

$$\Delta\alpha = \frac{1}{2} (\Delta\alpha_1 + \Delta\alpha_2) = -\frac{1}{4} \frac{k_2 \vec{B}_\parallel^2 I}{V_o} \times \sin 2\varepsilon [\cos 2\psi + \cos 2(90^\circ + \psi)] = 0. \quad (5)$$

Hence the angle  $\alpha$  is

$$\alpha = \frac{1}{2} (\alpha_1 + \alpha_2), \quad \alpha_i = \frac{1}{2} \frac{V_i - V'_i}{V_o} \quad (i = 1, 2). \quad (6)$$

### 3. Calibration and test of Hall probes and tilt sensors

#### 3.1. Planar Hall effect

For the radial magnetic field measurements we chose general-purpose transverse Hall probes BH-206 from F.W. Bell because of their high sensitivity, low temperature coefficients and low

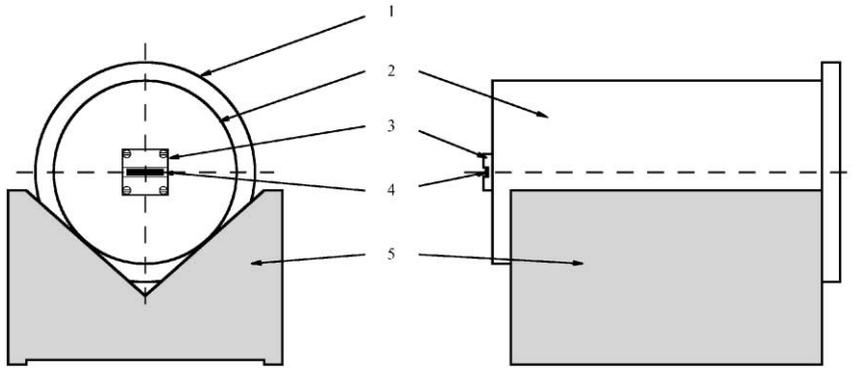


Fig. 2. Fixture used for Hall probe calibration, front and side views: (1) handle; (2) solid aluminum cylinder; (3) aluminum plate; (4) Hall probe; (5) aluminum base.

cost. Electrolytic tilt sensors RG33A from Spectron Systems Technology Inc. were chosen because of their high resolution of 0.1 arcsec ( $\approx 0.5 \mu\text{rad}$ ) within a range of  $\pm 0.25^\circ$  from the horizontal.

The Hall probe parameters  $V_o$  and  $k_2$  were measured in a test magnet with  $B = 1.45 \text{ T}$ . Fig. 2 shows the setup for the measurement of the planar Hall coefficient  $k_2$ . Each Hall probe is glued onto a square aluminum plate,  $2 \times 2 \text{ cm}^2$ , with 4 holes used for installation. As seen in Figs. 1 and 2, the plate has a groove for a Hall probe, which allows installation of the Hall probe upside down, if needed. The plate is screwed to a solid aluminum cylinder which rests on an aluminum base and the cylinder can be rotated manually using a handle. Not shown in Fig. 2 are a protractor and a vernier from a standard designer's drafting machine, which are screwed to the handle and the base, respectively, and provide a measurement of the angle of rotation with a relative precision of  $\sim 0.1^\circ$ . The whole apparatus was installed in a test magnet providing a uniform vertical magnetic field of 1.45 T.

Fig. 3 shows the Hall probe output voltage as a function of the angle of rotation of one of the probes. A five parameter function  $f(x) = A_o + A_1 \cos(x + \phi_1) + k_2 \cos(2x + \phi_2)$  was used to fit the measurements. The parameter  $A_o$  includes the contribution from the vertical field component, in case the axis of rotation is not perfectly horizontal, as well as the Hall probe's intrinsic voltage offset. The parameter  $A_1$  is nonzero if the axis of rotation

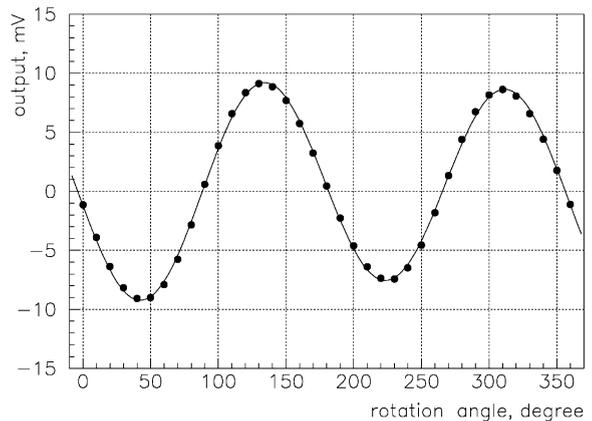


Fig. 3. Hall probe output voltage as a function of rotation angle. The dots are measurements and the line is the fitting function.

is not exactly perpendicular to the plane of the Hall probe. The agreement between measurement and fitting function  $f(x)$  was good for all Hall probes tested. To measure the parameter  $V_o$ , the cylinder was simply taken off the base and set on its handle in the same magnet. Several Hall probes with nearly the same parameters  $V_o$  and  $k_2$  were chosen for the radial magnetic field measurement.

### 3.2. Linearity

The linearity of the Hall probes and tilt sensors at small angles was measured in the test magnet

using the simple apparatus shown in Fig. 4. The platform with Hall probes and tilt sensors was installed on an aluminum channel ( $\sim 2$  m long) which can be tilted with respect to another aluminum channel firmly attached to the bottom pole of the magnet. The tilt can be changed manually using a screw at the end of the channel. A dial indicator was used to measure the relative gap between the two channels and hence the tilt angle.

Fig. 5 shows the results of these measurements for one of the Hall probes and tilt sensors. For convenience the voltage outputs are shown as differences between consecutive measurements,

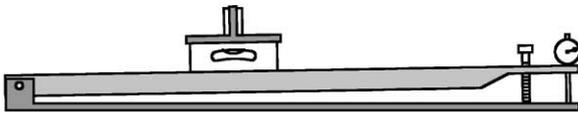


Fig. 4. Scheme of the Hall probe and tilt sensor calibration.

which were done in steps of about  $66 \mu\text{rad}$  in the tilt angle ( $0.005$  in. in the dial indicator). Fig. 5 indicates a very good linearity between tilt angle and Hall probe output voltage (a), but significant nonlinearities in the tilt sensor output voltage (b). We believe that the ripples seen in Fig. 5(a) are due to random variations of the actual tilt step size, which cannot be controlled very accurately. Assuming perfect linearity of the Hall probe, one can use the plot in Fig. 5(a) to make a correction for the variable step size. Fig. 5(c) shows the result for the tilt sensor measurements, corrected for the step size error. It is smoother than the plot in the Fig. 5(b).

As was mentioned above, we use the tilt sensor only to adjust the apparatus to the same angle  $\delta$  before and after  $180^\circ$  rotation. Thus nonlinearity of the tilt sensor is largely irrelevant. Still, we use the calibration chart in Fig. 5 for small corrections due to possible misadjustment of  $\delta$ .

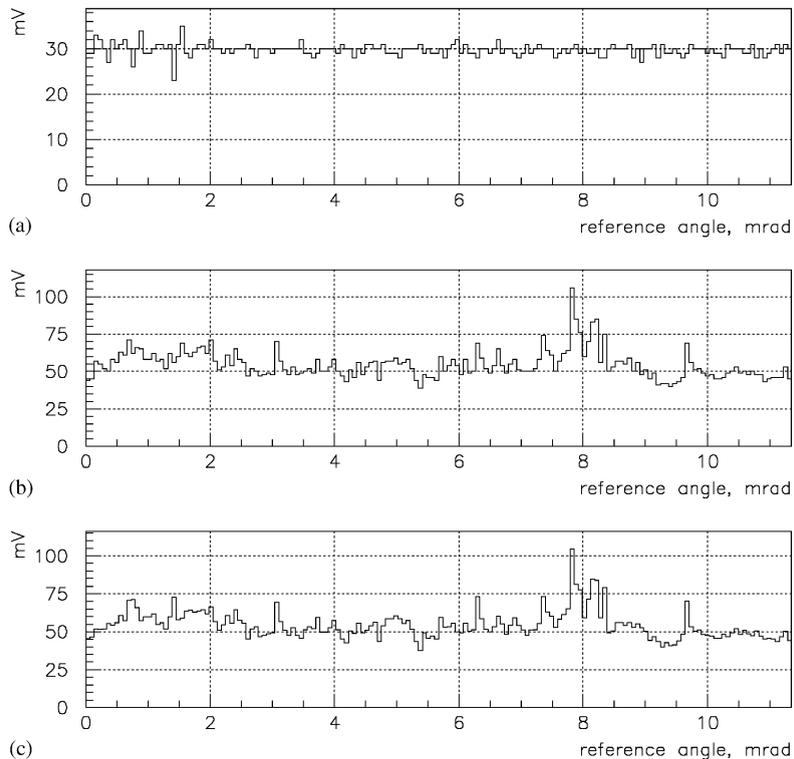


Fig. 5. Hall probe and tilt sensor outputs (differential) as functions of the reference angle: (a) Hall probe voltage; (b) tilt sensor voltage; (c) tilt sensor voltage corrected for the step size.

#### 4. Testing the assembled device

The assembled device for the radial magnetic field measurement, as described in Section 2, was tested in the test magnet. We measured the angle  $\alpha$  between the magnetic field and the vertical axis for 11 different tilts  $\delta$  over the whole range of the tilt sensor, which is about  $\pm 0.25^\circ$ . No dependence of  $\alpha$  on  $\delta$  was observed. Averaging all the results for the Hall probes #1 and #2 gave

$$\alpha_1 = 1894.7 \mu\text{rad}, \quad \alpha_2 = 1918.3 \mu\text{rad} \quad (7)$$

with an RMS value of  $4.5 \mu\text{rad}$  for both [5]. The difference  $\alpha_1 - \alpha_2 = -23.6 \mu\text{rad}$  was attributed to the planar Hall effect and the final result was taken to be  $\alpha = (\alpha_1 + \alpha_2)/2 = 1906.5 \mu\text{rad}$ .

Later, in the (g-2) magnet we studied directly the influence of the planar Hall effect on measurements with the assembled device by tilting the whole device in the azimuthal direction using a different number of shims under one of the edges of the device. Fig. 6 shows the angles  $\alpha_1$  and  $\alpha_2$  as functions of the number of shims. The dependences are linear as expected but have different slopes (1 : 2.3), which do not match the ratio of the measured planar Hall coefficients for these probes and therefore cannot be explained by the planar Hall effect alone.

This disagreement was caused by an additional source of systematic error associated with  $\varepsilon \neq 0$  (declination of the axis of  $180^\circ$  rotation from vertical in the azimuthal direction), namely a misalignment of the Hall probe plane with respect to the aluminum support. Fig. 7 shows the position of a misaligned Hall probe before and after the  $180^\circ$  rotation of the circular support with respect to the base. If  $\beta \neq 0$  is the misalignment angle, the strong vertical field will contribute to the radial field measurement. According to Eq. (1) this contribution is  $C_1 \sin \beta \sin \varepsilon$ , whereas the planar Hall effect is  $C_2 \sin 2\varepsilon$ . For  $\varepsilon \ll 1$  the net effect is  $(C_1 \sin \beta + 2C_2)\varepsilon$ , and hence Eq. (6) should be corrected

$$\alpha = a\alpha_1 + b\alpha_2, \quad \text{where } a + b = 1. \quad (8)$$

For our assembled device  $a/b$  is 2.3 and hence  $a = 0.697$  and  $b = 0.303$ . For the radial field measurements in the test magnet, discussed above, Eq. (8) gives

$$\begin{aligned} \alpha &= 0.697 \cdot 1894.7 \mu\text{rad} + 0.303 \cdot 1918.3 \mu\text{rad} \\ &= 1901.9 \mu\text{rad} \end{aligned} \quad (9)$$

which differs from  $(\alpha_1 + \alpha_2)/2$  by  $4.6 \mu\text{rad}$ .

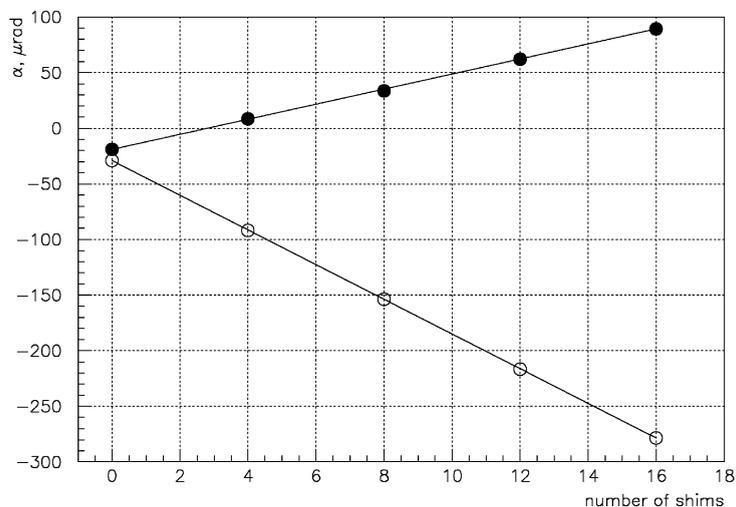


Fig. 6. Measurement of the angle  $\alpha$  by Hall probe #1 (closed circles) and by Hall probe #2 (open circles).

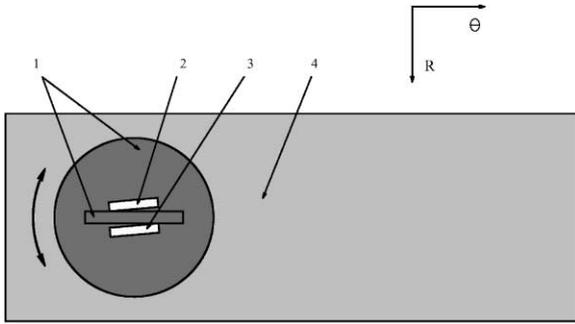


Fig. 7. Possible Hall probe misalignment: (1) aluminum support; (2) Hall probe initial position; (3) Hall probe position after the 180° rotation; (4) aluminum base. Top view. For side and front views of the support and other parts see Fig. 1.

## 5. Measurements in the (g-2) magnet

Several important tests for the Hall device were made in the muon storage region in the course of studying and shimming the magnetic field in the (g-2) magnet. The NMR system [2] measured the magnetic field with an accuracy at the sub-ppm level. While only the absolute value of the field is measured, still useful information about the variation of the radial field component is obtained.

The (g-2) magnet is a superferric storage ring magnet with a C-shaped cross-section, open towards the ring center. The tapered pole pieces are 56 cm wide at the base and the vertical gap is 18 cm. The muon storage region with its 9 cm diameter aperture is located at the center of the gap and has a toroidal shape with a radius for the central orbit of 711.2 cm. Since the transverse dimensions of the storage region are much less than the radius of the central orbit, and the magnet is highly uniform in azimuth, the magnetic field in the muon storage region can be considered as two-dimensional with vertical and radial components only. From the Poisson equation, a two-dimensional magnetic field can be decomposed into a series of multipoles

$$B_X = \sum_{n=0}^{\infty} C_n r^n \cos(n\phi) - \sum_{n=0}^{\infty} D_n r^n \sin(n\phi)$$

$$B_Y = \sum_{n=0}^{\infty} C_n r^n \sin(n\phi) + \sum_{n=0}^{\infty} D_n r^n \cos(n\phi) \quad (10)$$

where  $B_Y$  and  $B_X$  are the vertical and radial components of the field, respectively, and  $r$  and  $\phi$  are polar coordinates:  $X = r \cos \phi$ ,  $Y = r \sin \phi$  in the two-dimensional geometry, which we will use from now on.

For the (g-2) magnet the radial field in the muon storage volume is less than  $10^{-4}$  of the vertical field, thus

$$|\vec{B}| = \sqrt{B_Y^2 + B_X^2} \approx |B_Y| + \frac{B_X^2}{2|B_Y|}$$

$$= |B_Y| \times (1 + O(10^{-8})). \quad (11)$$

Hence the NMR probes measure essentially  $B_Y$ . With several probes in the  $X$ - $Y$  plane we can determine several low-order coefficients  $C_n$  and  $D_n$  in Eq. (10) but not  $D_0$ , which is the radial field at the center of the muon storage region. However the NMR measurements determine the variation of the radial magnetic field as a function of  $X$  and  $Y$ . Measurement of  $D_0$  is the main objective for our Hall probe device.

Fig. 8 shows the radial field reconstructed from NMR measurements (solid lines), and measured directly by the Hall probe device (dots). Since the coefficient  $D_0$  cannot be extracted from the NMR measurements, it was taken from the Hall probe measurement at  $X = Y = 0$ . The right-hand plots in Fig. 8 are for a case where the magnetic field was perturbed by removal of the shims on the pole edges, and the left-hand plots are obtained from measurements with the shims in place. In both cases the agreement between Hall probe and NMR measurements is good. The NMR probes were located within a circle  $\sqrt{X^2 + Y^2} \leq 4.5$  cm and the accuracy of the field reconstruction beyond this circle is poorer. For the field with large gradients (right plots), the precision of the field measurements are limited by the spatial accuracy of the location of the NMR probes ( $\sim 0.5$  mm) and of the Hall probes ( $\sim 3$  mm). For the field with shims in place (left plots), the differences between NMR and Hall probe measurements are well within 10 ppm.

The Hall probe device was used extensively to measure the radial magnetic field in the muon (g-2) storage ring prior to the 1997 and 1998 data taking runs. Measurements were taken at the center of

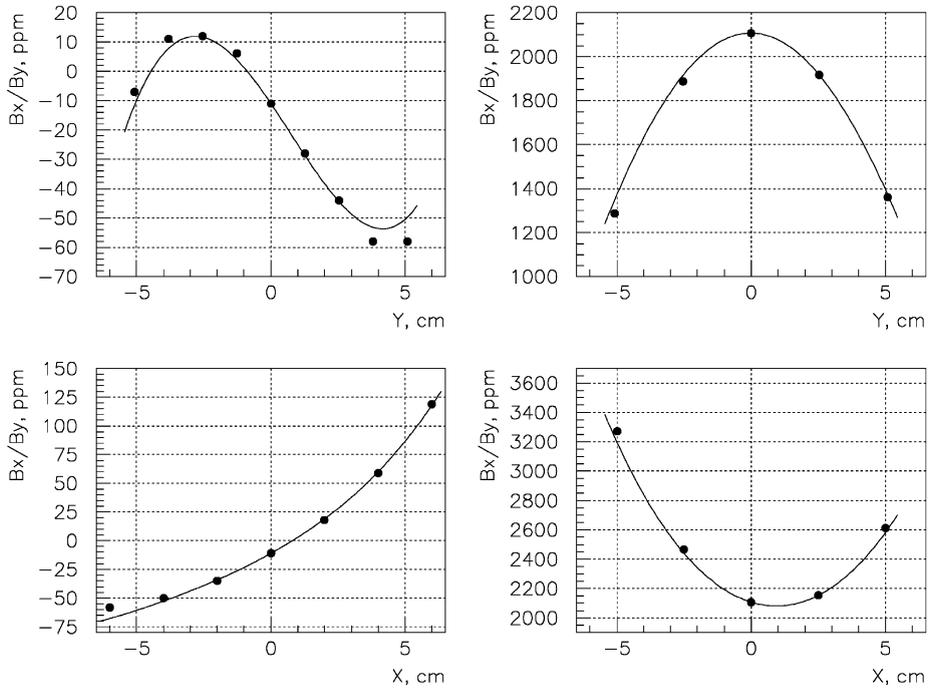


Fig. 8. Radial magnetic field in parts per million with respect to vertical field.

each of the 36 pole sections of the magnet. It took about 12 hours to complete the entire cycle of measurements. The two measurements at the same point at the beginning and end of the cycle always agreed to within 5–10 ppm with respect to the vertical field.

The radial magnetic field, measured at the center of the pole sections, typically varied by 200 ppm peak to peak over the ring, and the ring average was 20 ppm for the 1997 run and –20 ppm for the 1998 run. This difference was due to additional field shimming done between the runs. The corresponding vertical shifts of the muon beam center were 1 mm and –1 mm, respectively, and hence slightly reduced the muon storage efficiency.

The shimming for the magnet includes correction coils which allow adjustment of the average radial magnetic field. The correction coils are printed circuit (PC) boards glued to the surfaces of the poles which are  $10^\circ$  wide. Each PC board has 120 etched azimuthal strips, equally spaced by 0.1 in. in the radial direction. Strips from adjacent PC boards are connected by pin connectors, forming

120 conductive rings on both the top and bottom poles. A uniform radial magnetic field can be created by powering all coils with same current but opposite direction for the top and bottom coils. For infinitely wide poles one can calculate analytically the radial field  $\Delta B_X$  created by such currents. Thus for a current of 0.2 A through each of the wires we find

$$\begin{aligned} \Delta B_X &= \mu_o \frac{\Delta I}{\Delta X} = 4\pi \times 10^{-7} \text{ T m/A} \times \frac{0.2\text{A}}{2.54 \times 10^{-3}\text{m}} \\ &= 9.89 \times 10^{-5} \text{ T} \end{aligned} \quad (12)$$

$$\begin{aligned} \Delta B_X/B_Y &= 9.89 \times 10^{-5}/1.45 \\ &= 6.82 \times 10^{-5} = 68.2 \text{ ppm.} \end{aligned} \quad (13)$$

Numerical calculations for the real geometry and estimates based on NMR measurements vary from 56 to 62 ppm. The radial field effect from such a current distribution was measured with the Hall probe device as the difference of two measurements, before and after powering the coils with a current of 0.2 A. The result was 51 ppm, which is close to the expected value.

Muon storage optimization with correction coils and other beam dynamics studies provided independent estimates for the average radial magnetic field:  $20 \pm 10$ ,  $-10 \pm 10$ , and  $-20 \pm 10$  ppm for 1997, 1998 and 1999, respectively. These results confirmed our direct measurements with the Hall probe device. The results for the 1998 and 1999 runs indicate the good mechanical stability of the (g-2) magnet.

## 6. Conclusion

Hall probes are been used quite extensively in high-energy physics for precision measurement of the direction of a magnetic field and for other applications for a long time. Accuracy of the best such measurements is typically of order  $10^{-4}$ , see Ref. [6].

We have built a Hall probe device for the measurement of the radial component of the magnetic field in the (g-2) magnet. A number of test measurements, both in a test magnet and in the (g-2) magnet, showed the accuracy of the device to be better than  $10^{-5}$  with respect to the vertical field and perhaps as good as  $5 \times 10^{-6}$ . This accuracy is quite adequate for the muon (g-2) experiment.

In a recent work [7] with an apparatus similar to ours, the authors have achieved further improvement in the precision measurement of minor components of a magnetic field.

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